Is there an alternative to dark matter? From large to small scales

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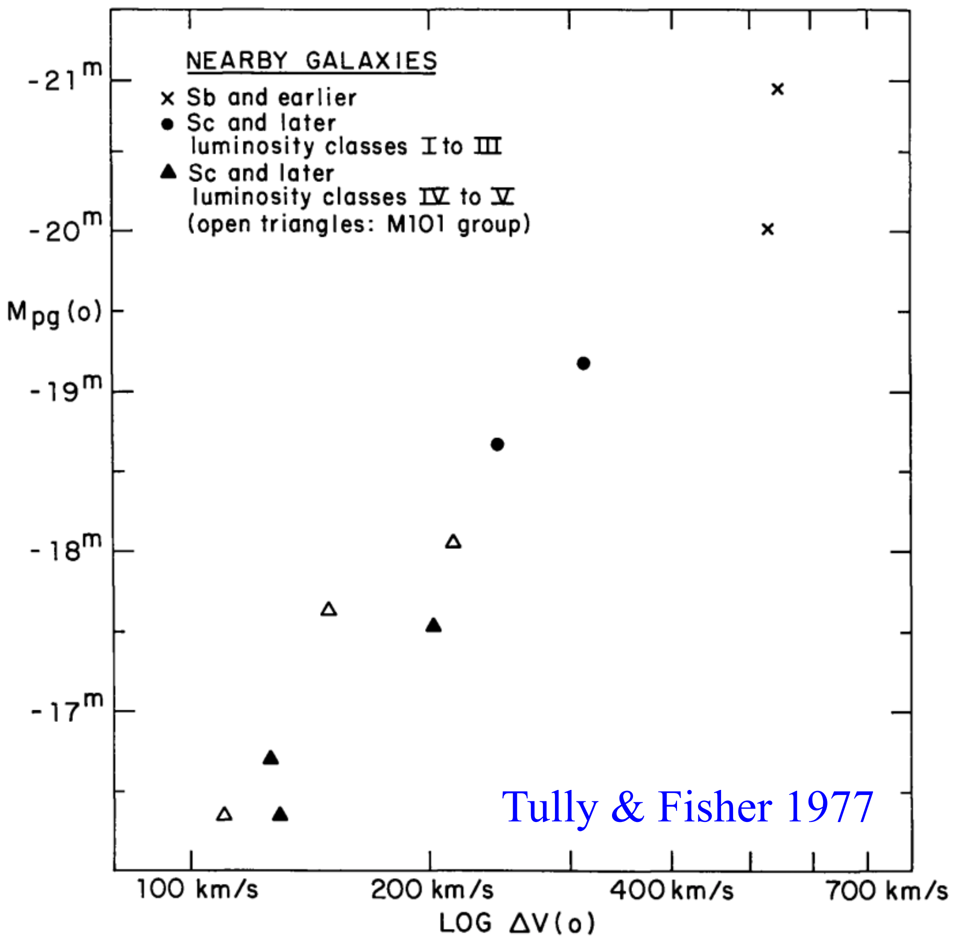
Regularities in the dynamics of galaxies in HI



$L \propto \Delta v^{\alpha}$

$\alpha = 2.5 - 4$

(slope of 6.25-10 in mag)

Half of the velocity width at 20% of the peak flux = proxy for rotational velocity

Regularities in the dynamics of galaxies in HI

Armed with this knowledge 41 years ago, Milgrom proposed his **MOND paradigm**, or just ***Milgrom's relation***:

-If observed RCs are flat, then gravity must effectively fall like 1/r
-The discrepancy sets in at different radii in different galaxies, so a more relevant scale than radius is the centripetal acceleration

$$\boxed{\begin{array}{ll} g = g_N & \text{if } g \gg a_0 \\ g = (g_N\, a_0)^{1/2} & \text{if } g \ll a_0 \end{array}}$$

MOND
Milgrom 1983

$a_0 \sim 10^{-10}\ \mathrm{m/s^2}$

$\Rightarrow$ Velocity predicted to be flat (until the external field dominates), and Tully-Fisher relation predicted to be a relation between the **total baryonic mass** of galaxies and the **asymptotic circular velocity**, with a **slope of 4**, with no dependence on secondary parameters such as baryonic surface density

Very strong, bold and unintuitive predictions at the time!

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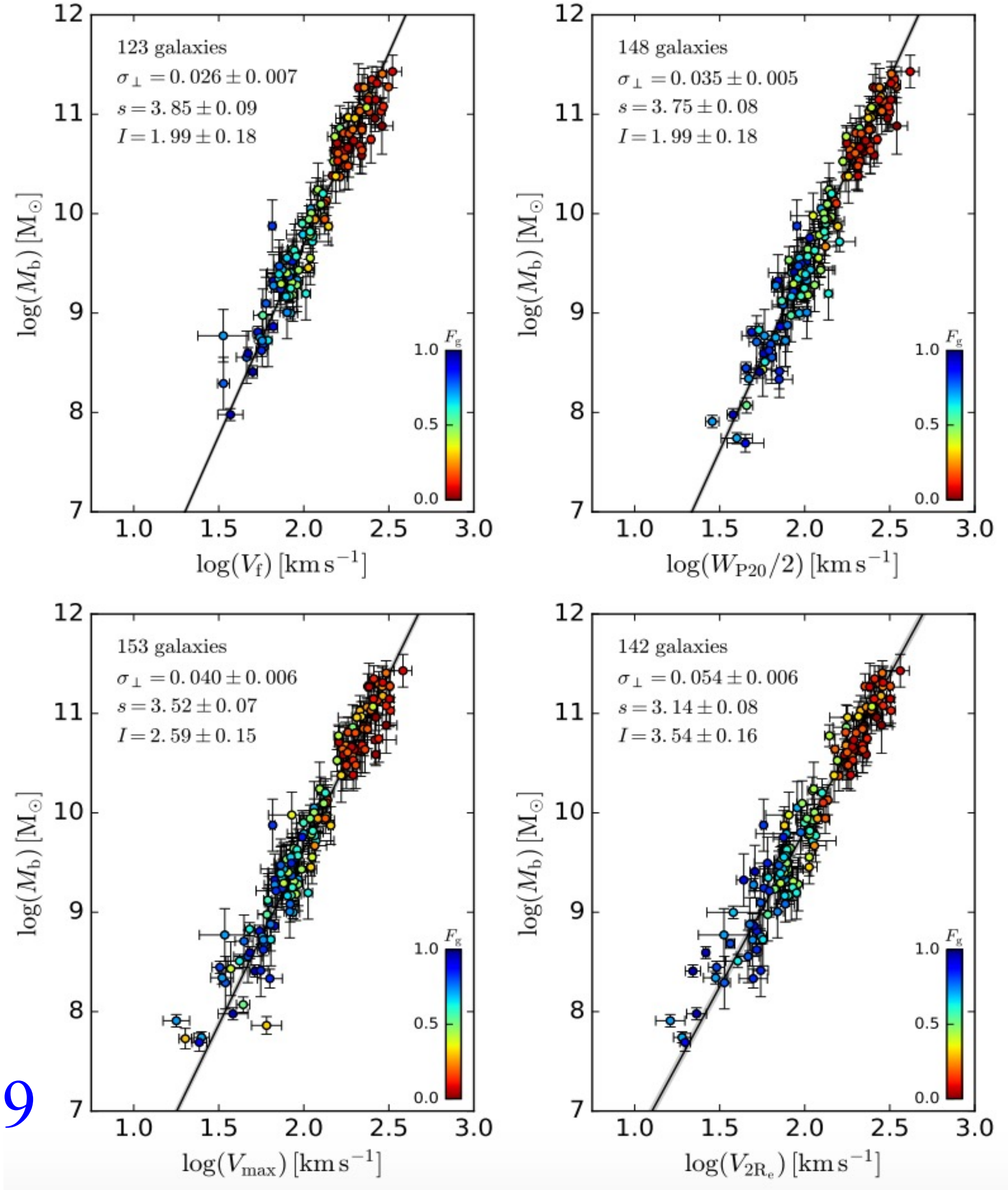
In Newtonian gravity, assume that one has a fixed factor between the total mass and the baryonic one at fixed baryonic mass M_b, one would then expect something like $V^4 \sim M_b^2/R^2 \sim M_b\,\Sigma_b$ where Σ_b is the surface density, but what is predicted is $V^4 \sim M_b$, with no dependence on size or Σ_b

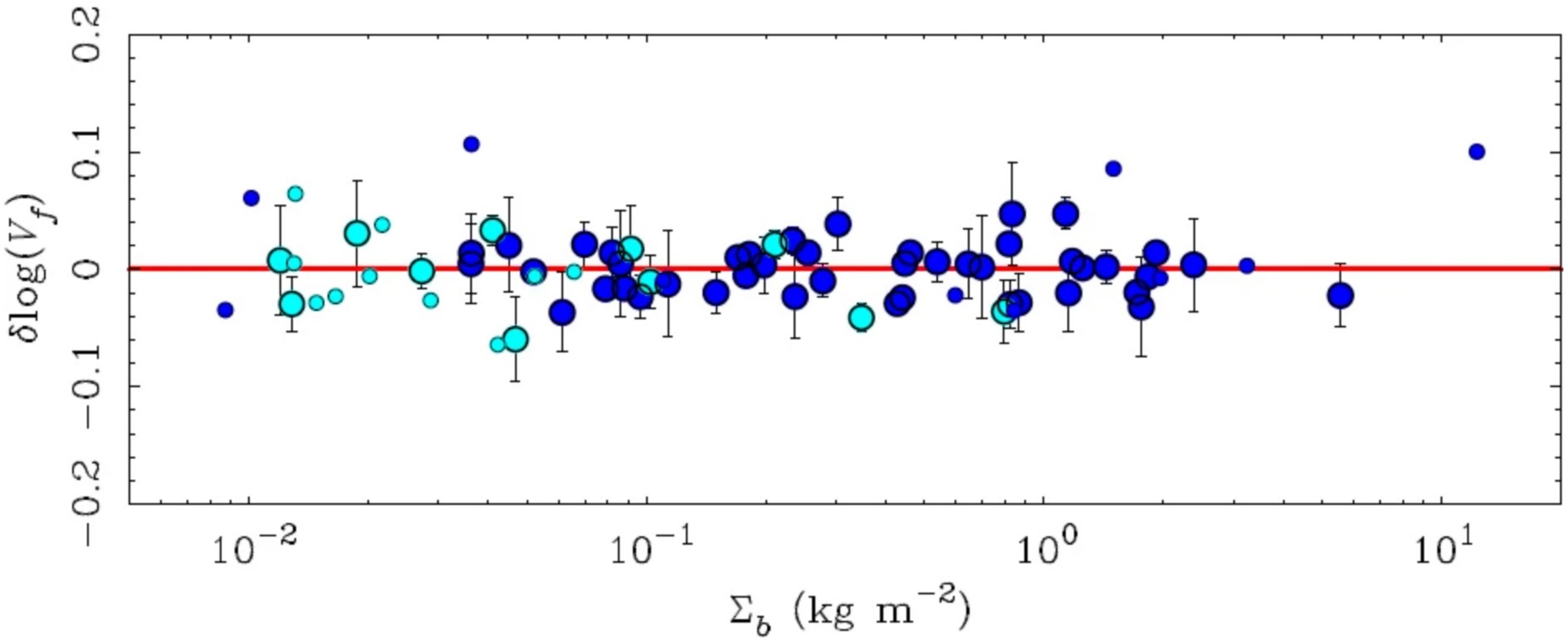
Verified prediction: Baryonic TF

- $\text{Log } M_b = \alpha \log V_f - \log \beta$
- $\alpha \approx 4$

- **Intrinsic scatter ~ 6%**

Lelli et al. 2019



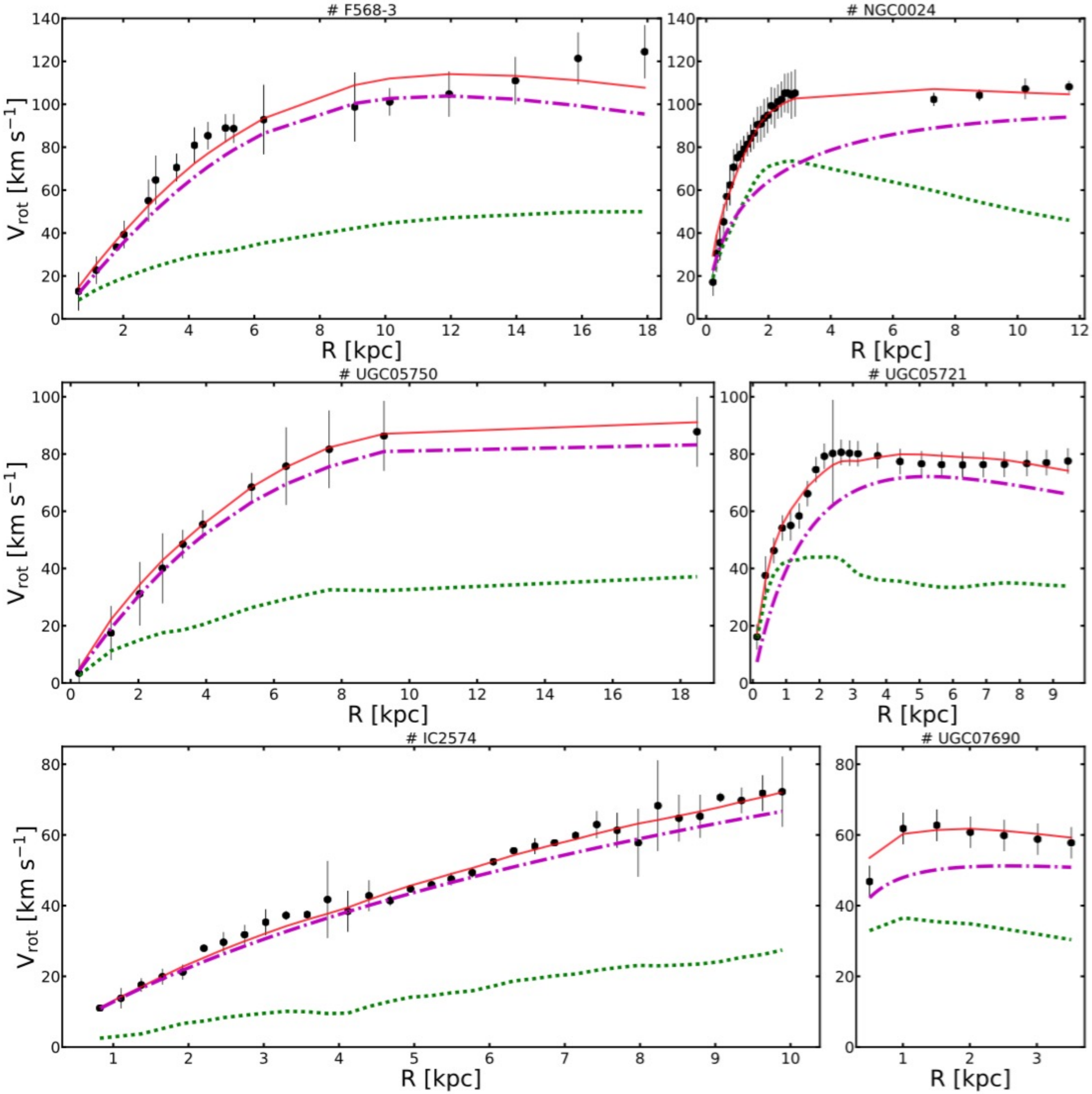


Famaey & McGaugh 2012

Diversity of rotation curves as a function of baryonic surface density

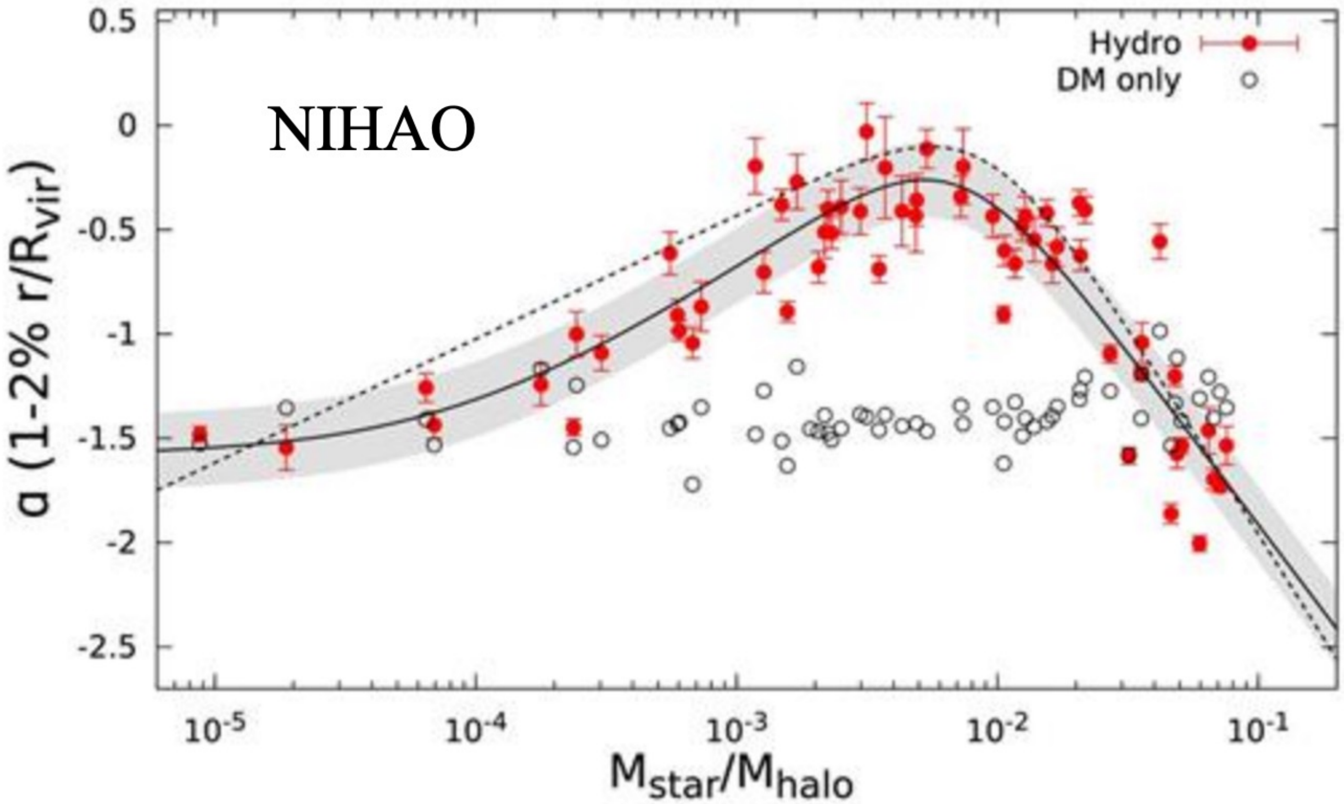
"We predict a correlation between the value of the average surface density of a galaxy and the steepness with which the rotational velocity rises to its asymptotic value."

Milgrom (1983)



Ghari, Famaey, et al. (2019)

Dark matter halos are (almost) a one-parameter family (driven by mass)



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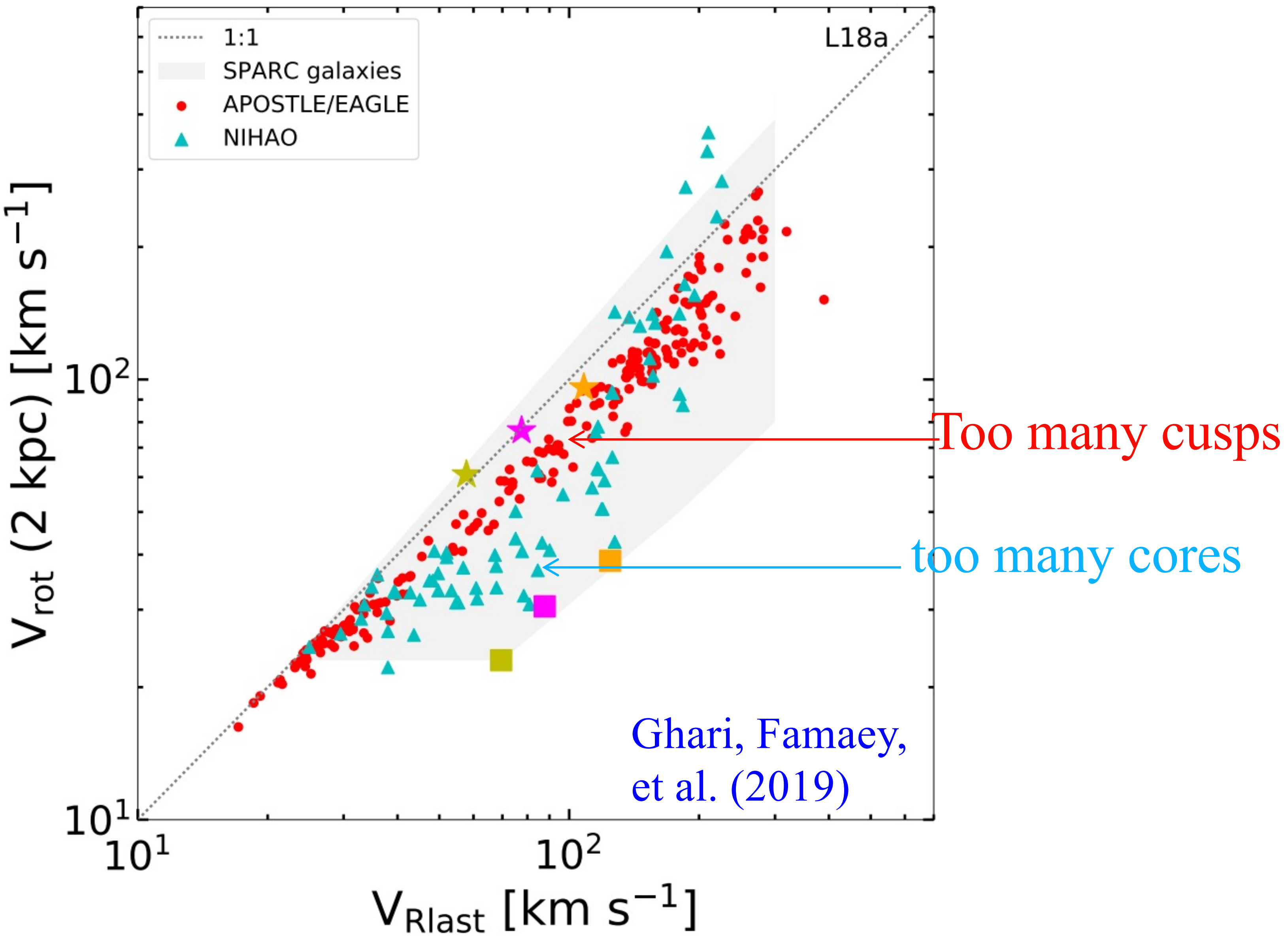

Modifying gravity ?

$a_N = \mu(a/a_0)\, a$

or for $x \ll 1$, one should have $\mu(x) \to x$ or $\nu(x) \to 1/\sqrt{x}$.

$a = \nu(a_N/a_0)\, a_N$,

Newtonian Lagrangian density in the non-relativistic limit : $\mathcal{L} = -\rho\left(\Phi - \frac{1}{2}\vec{v}\cdot\vec{v}\right) - \frac{1}{8\pi G}\mathcal{L}_N$

$$\mathcal{L}_N = \nabla\Phi\cdot\nabla\Phi \equiv (\nabla\Phi)^2 \quad \longrightarrow \quad \mathcal{L}_{\mathrm{AQUAL}} = a_0^2\mathcal{F}((\nabla\Phi)^2/a_0^2),$$

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$$\mathcal{F}(Y) \to Y \text{ for } Y \gg 1 \text{ and } \mathcal{F}(Y) \to \frac{2}{3}Y^{3/2} \text{ for } Y \ll 1$$

$$\mathcal{F}'(Y) = \mu(\sqrt{Y}) \quad \longrightarrow \quad \nabla \cdot (\mu(|\nabla\Phi|/a_0)\nabla\Phi) = 4\pi G\rho$$

$\nabla^2\Phi = 4\pi G(\rho - \nabla \cdot \vec{\Pi})$ with $\vec{\Pi} = -(\chi\vec{a})/(4\pi G)$ and $\mu(a/a_0) = 1 + \chi(a/a_0)$

Modifying gravity ?

$$\mathcal{L}_{\mathrm{N}} = \nabla\Phi \cdot \nabla\Phi \equiv (\nabla\Phi)^2 \quad \longrightarrow \quad \mathcal{L}_{\mathrm{QUMOND}} = 2\nabla\Phi \cdot \nabla\Phi_{\mathrm{N}} - a_0^2 \mathcal{Q}((\nabla\Phi_{\mathrm{N}})^2/a_0^2)$$

$$\nabla^2\Phi = \nabla \cdot (\nu(|\nabla\Phi_{\mathrm{N}}|)\,\nabla\Phi_{\mathrm{N}}) \quad \longrightarrow \quad \rho_{\mathrm{ph}}^{\mathrm{QUMOND}} = \frac{\nabla \cdot (\bar{\nu}(|\nabla\Phi_{\mathrm{N}}|/a_0)\,\nabla\Phi_{\mathrm{N}})}{4\pi G}$$

where $\bar{\nu} = \nu - 1$

=> very convenient for numerical solvers
(e.g., **Phantom of Ramses** patch of the RAMSES code)

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Note that :

$\vec{a} = \nu(|\vec{a}_{\rm N}|/a_0)\vec{a}_{\rm N}$ can only be exact where $\nabla|\nabla\Phi_{\rm N}| \times \nabla\Phi_{\rm N} = 0$

$\vec{d^{Q}} = \nu\nabla\Phi_{\rm N} - \nabla\Phi$ with $\nabla\times\vec{d^{Q}} = \nabla\nu\times\nabla\Phi_{\rm N} = \nu'\nabla|\nabla\Phi_{\rm N}|\times\nabla\Phi_{\rm N}$

Exact (and approximate) solutions for disks

$$\Sigma_{\rm K}(R) = Mh/[2\pi(R^2 + h^2)^{3/2}] \qquad \text{(the Kuzmin disk)}$$

$$\Phi_{\rm K,N} = -GM/[r^2 + (|z|^2 + h)^2]^{1/2}$$

$$\text{Outside of the disk}: \vec{a} = \nu(|\vec{a}_{\rm N}|/a_0)\vec{a}_{\rm N}$$

$$\text{with} \quad \vec{a}_{\rm N}(R,h) = GM(\vec{R} \pm \vec{h})/|\vec{R} \pm \vec{h}|^3$$

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with $\vec{a}_{\rm N}(R,h) = GM(\vec{R} \pm \vec{h})/|\vec{R} \pm \vec{h}|^3$

$$\Phi_{\rm K}(R,z) = \sqrt{GMa_0}\,\log(R^2 + [|z| + h]^2)/2 \quad \text{in deep-MOND}$$

In the disk :

$$V^2 = \sqrt{GMa_0}R^2/(R^2 + h^2) \neq \sqrt{GMa_0}R^{3/2}/(R^2 + h^2)^{3/4}$$

$$a = \nu(a_N^+/a_0)\,a_{\rm N} \qquad \text{with} \qquad a_N^+ \equiv \left(a_N^2 + (2\pi G\Sigma)^2\right)^{1/2}$$

External field effect

$$\nabla \cdot \left(|\nabla \Phi_{\rm int} - \vec{a}_{\rm ext}| \left(\nabla \Phi_{\rm int} - \vec{a}_{\rm ext} \right) \right) = 4\pi G a_0 \rho$$

when $a_{int} << a_{ext} << a_0$, back to Newton with renormalization of G

$$G \rightarrow G/\mu \text{ and } r \rightarrow (1 + \log'(\mu) \sin^2(\theta))^{1/2} r$$

in AQUAL

$$G \rightarrow G\nu \text{ and } r \rightarrow r/(1 + \log'(\nu) \sin^2(\theta)/2)$$

in QUMOND

$$\eta = \nu_e \left(1 + \frac{K_e}{3} \right), \quad K_e \equiv \frac{\partial \ln \nu_e}{\partial \ln g_{N,e}}$$

External field effect

$$\nabla \cdot \left(\left| \nabla \Phi_{\text{int}} - \vec{a}_{\text{ext}} \right| \left(\nabla \Phi_{\text{int}} - \vec{a}_{\text{ext}} \right) \right) = 4\pi G a_0 \rho$$

when $a_{int} << a_{ext} << a_0$, back to Newton with renormalization of G

$$G \to G/\mu \text{ and } r \to (1 + \log'(\mu) \sin^2(\theta))^{1/2} r \qquad G \to G\nu \text{ and } r \to r/(1 + \log'(\nu) \sin^2(\theta)/2)$$

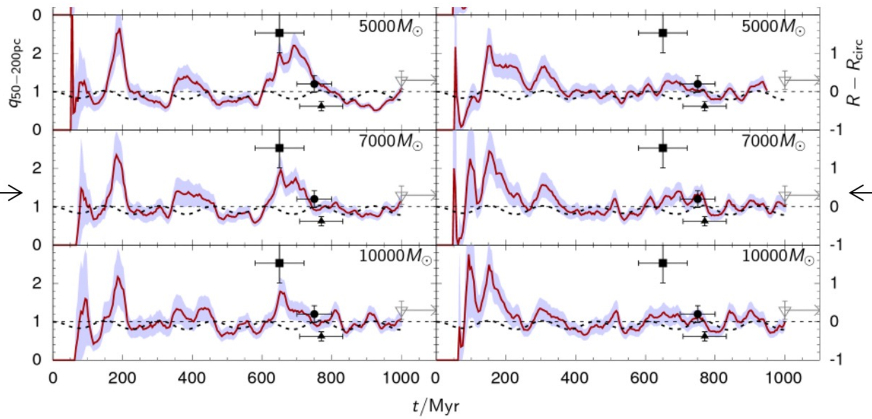
in AQUAL

$$\eta = \nu_e \left(1 + \frac{K_e}{3}\right), \quad K_e \equiv \frac{\partial \ln \nu_e}{\partial \ln g_{N,e}}$$

in QUMOND

Kroupa et al. (2022) : **denser** leading tails (50-200 pc) of open clusters close to pericenter, e.g. Hyades, Coma Berenices, NGC 752

MOND ⟶



⟵ Newton

External field effect

Table 1. Summary of Fitting Functions of the EFE-dependent RAR for the Outer Part of Rotation Curves

case	theory	$\frac{g_{\rm MOND}}{g_{\rm N}}(y,e)$ for $y \equiv \frac{g_{\rm N}}{a_0}$ and $e \equiv \frac{g_{\rm ext}}{a_0}$ (or $e_{\rm N} \equiv \frac{g_{\rm N,ext}}{a_0}$)	references
(0) algebraic MOND (no EFE)	generic	$\nu(y)$ $(e=0)$	M83
(1) asymptotic limit	AQUAL	$\frac{1}{\mu(e)}\left(1+\hat{\mu}(e)-\hat{\mu}(e)\frac{\sin^2\theta}{2}\right)^{-1/2}$	M10, C21
(2) asymptotic limit	QUMOND	$\nu(e_{\rm N})\left(1+\frac{\hat{\nu}(e_{\rm N})}{2}-\hat{\nu}(e_{\rm N})\frac{\sin^2\theta}{4}\right)$	M10, C21
(3) 1D analytic	generic	$\frac{1}{2}+\sqrt{\left(\frac{1}{2}-e\frac{A_e}{y}\right)^2+\frac{B_e}{y}}-e\frac{A_e}{y}$	FM12, C21
(4) Freundlich-Oria analytic	QUMOND	$\nu\left(\min\left[y+\frac{e_{\rm N}^2}{3y}, e_{\rm N}+\frac{y^2}{3e_{\rm N}}\right]\right)$	F21, O21
(5) QUMOND numerical	QUMOND	$\nu(y_1)\left[1+\tanh\left(\frac{0.825e_{\rm N}}{y}\right)^{3.7}\frac{\hat{\nu}(y_1)}{3}\right]$	Z21
(6) AQUAL numerical	AQUAL	$\nu(y_\beta)\left[1+\tanh\left(\frac{\beta e_{\rm N}}{y}\right)^{\gamma}\frac{\hat{\nu}(y_\beta)}{3}\right]$	this work
(7) AQUAL numerical	AQUAL	$\frac{2\sqrt{2}}{3\sqrt{y}}[1+0.56\exp(3.02\log_{10}(y))]^{0.368}10^{2F_e}$	H19

Chae & Milgrom 2022

Generalizations with more dimensioned constants

$$\mathcal{L}_{\mathrm{EMOND}} = \Lambda(\Phi)\,\mathcal{F}((\nabla\Phi)^2/\Lambda(\Phi)),$$

$$\mathcal{L}_{\mathrm{GQUMOND}} = 2\nabla\Phi \cdot \nabla\Phi_{\mathrm{N}} - a_0^2\mathcal{P}(\Phi_{\mathrm{N}}, \nabla\Phi_{\mathrm{N}}, \nabla^2\Phi_{\mathrm{N}}, \ldots, \nabla^n\Phi_{\mathrm{N}})$$

$$\mathcal{L}_{\mathrm{TriMOND}} = 2\nabla\Phi \cdot \nabla\Phi_{\mathrm{N}} - a_0^2\mathcal{P}((\nabla\Phi_{\mathrm{N}})^2/a_0^2, (\nabla\varphi)^2/a_0^2, (2\nabla\Phi_{\mathrm{N}} \cdot \nabla\varphi)/a_0^2)$$

Global fit to galaxy rotation curves

$$\nu_n(x) = \left[\frac{1+\left(1+4x^{-n}\right)^{1/2}}{2}\right]^{1/n},$$

$$\nu_\delta(x) = \left(1-e^{-x^{\delta/2}}\right)^{-1/\delta},$$

$$\nu_\gamma(x) = \left(1-e^{-x^{\gamma/2}}\right)^{-1/\gamma} + (1-1/\gamma)\, e^{-x^{\gamma/2}}$$

$$a_{\mathrm{N}}(R) = \Upsilon_{\mathrm{gas}} \operatorname{sign}\left(V_{\mathrm{gas}}\right) \frac{V_{\mathrm{gas}}^2}{R} + \Upsilon_{\mathrm{disk}} \frac{V_{\mathrm{disk}}^2}{R} + \Upsilon_{\mathrm{bul}} \frac{V_{\mathrm{bulge}}^2}{R}$$

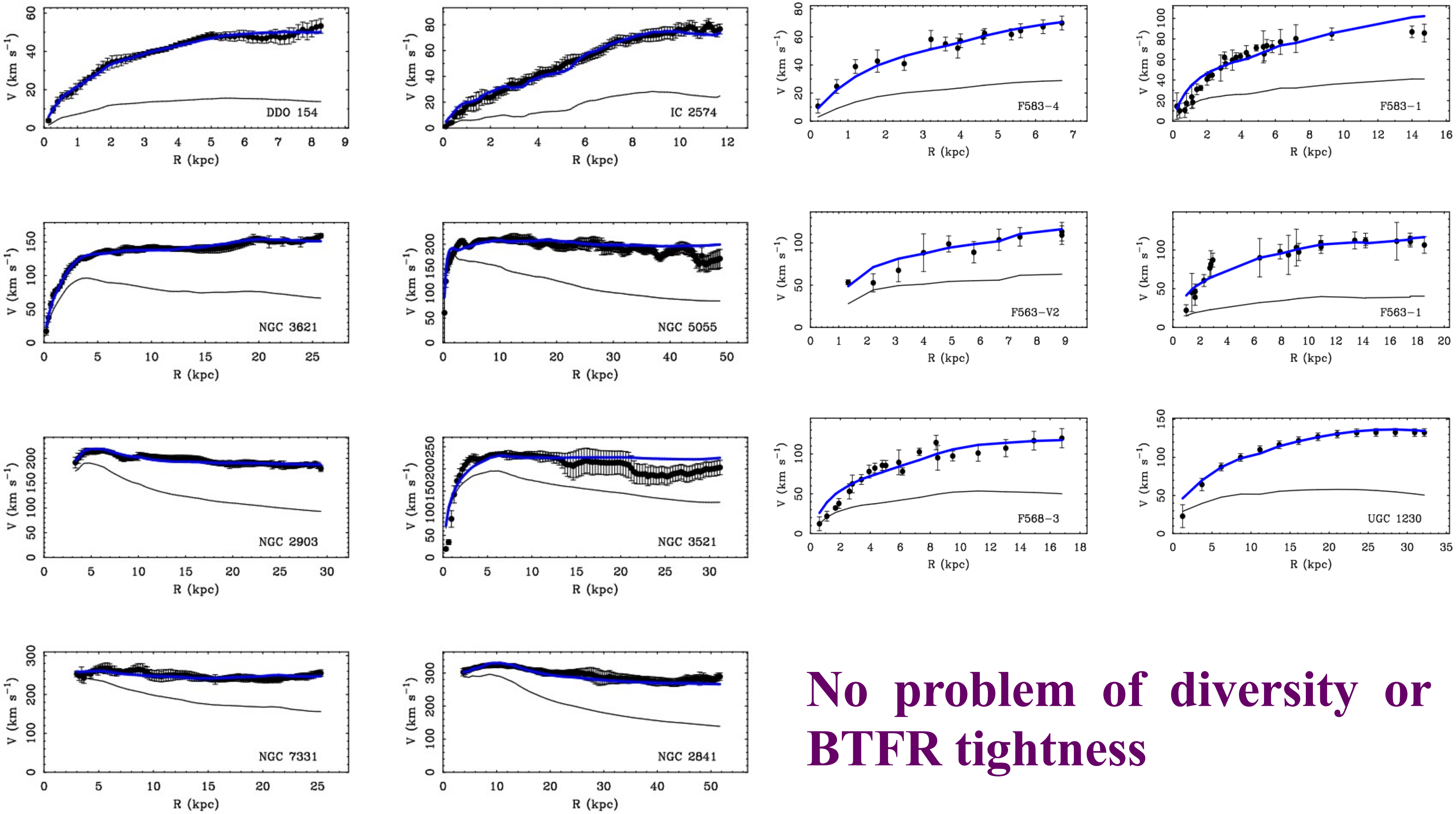
=> $$V(R\alpha)\sin i = \sin i_{\mathrm{fit}} \sqrt{a(R)R\alpha}.$$

Desmond, Hees & Famaey (2024)

Global fit to galaxy rotation curves

IF family	EFE model	shape	a_0	e_N	Q_2	σ_{Q2}	$\alpha_{\rm grav}$	ΔBIC	ΔBIC(P)
RAR IF	No EFE	1	$1.03^{+0.03}_{-0.03}$	—	$29.2^{+0.3}_{-0.4}$	8.7	$0.74^{+0.03}_{-0.03}$	0	0
δ	No EFE	$0.97^{+0.04}_{-0.04}$	$1.02^{+0.04}_{-0.04}$	—	$29.4^{+0.4}_{-0.5}$	8.7	$0.78^{+0.06}_{-0.06}$	11.1	−10.2
δ	AQUAL global	$0.98^{+0.04}_{-0.04}$	$1.03^{+0.04}_{-0.04}$	$0.0017^{+0.001}_{-0.001}$	$29.4^{+0.4}_{-0.5}$	8.7	$0.77^{+0.07}_{-0.06}$	18.7	15.2
δ	AQUAL local	$1.14^{+0.05}_{-0.05}$	$1.25^{+0.05}_{-0.05}$	$\mathit{0.0048}^{+0.0082}_{-0.0020}$	$30.2^{+0.5}_{-0.5}$	8.9	$0.71^{+0.06}_{-0.06}$	1090	1480
δ	QUMOND global	$0.98^{+0.04}_{-0.04}$	$1.03^{+0.04}_{-0.04}$	$0.0049^{+0.0015}_{-0.0019}$	$29.4^{+0.4}_{-0.5}$	8.7	$0.77^{+0.07}_{-0.06}$	13.9	−1.98
δ	QUMOND local	$1.08^{+0.05}_{-0.05}$	$1.17^{+0.04}_{-0.04}$	$\mathit{0.0050}^{+0.0030}_{-0.0018}$	$29.9^{+0.5}_{-0.5}$	8.8	$0.73^{+0.07}_{-0.06}$	1090	1470
γ	No EFE	$1.03^{+0.07}_{-0.07}$	$1.03^{+0.03}_{-0.03}$	—	$29.1^{+0.4}_{-0.4}$	8.6	$0.71^{+0.07}_{-0.07}$	5.93	−1.99
γ	AQUAL global	$1.04^{+0.07}_{-0.07}$	$1.04^{+0.03}_{-0.03}$	$0.0018^{+0.0010}_{-0.0010}$	$29.2^{+0.4}_{-0.4}$	8.7	$0.71^{+0.07}_{-0.07}$	14.7	14.9
γ	AQUAL local	$1.14^{+0.06}_{-0.06}$	$1.19^{+0.04}_{-0.04}$	$\mathit{0.0048}^{+0.0070}_{-0.0020}$	$30.8^{+0.4}_{-0.4}$	9.2	$0.76^{+0.06}_{-0.06}$	1100	1510
γ	QUMOND global	$1.04^{+0.07}_{-0.07}$	$1.04^{+0.03}_{-0.03}$	$0.0050^{+0.0015}_{-0.0018}$	$29.2^{+0.4}_{-0.4}$	8.6	$0.71^{+0.07}_{-0.06}$	9.69	8.32
γ	QUMOND local	$1.11^{+0.07}_{-0.07}$	$1.14^{+0.04}_{-0.04}$	$\mathit{0.0050}^{+0.0030}_{-0.0017}$	$30.2^{+0.4}_{-0.4}$	9.0	$0.73^{+0.06}_{-0.06}$	1080	1490
n	No EFE	$1.02^{+0.04}_{-0.04}$	$1.08^{+0.04}_{-0.04}$	—	$28.4^{+0.4}_{-0.4}$	8.4	$0.87^{+0.07}_{-0.06}$	17.7	23.3
n	AQUAL global	$1.03^{+0.04}_{-0.04}$	$1.09^{+0.04}_{-0.04}$	$0.0018^{+0.0009}_{-0.0010}$	$28.4^{+0.4}_{-0.4}$	8.4	$0.86^{+0.07}_{-0.06}$	26.3	33.2
n	AQUAL local	$1.19^{+0.06}_{-0.04}$	$1.31^{+0.05}_{-0.05}$	$\mathit{0.0048}^{+0.0094}_{-0.0020}$	$29.4^{+0.5}_{-0.5}$	8.7	$0.79^{+0.07}_{-0.06}$	1100	1490
n	QUMOND global	$1.03^{+0.04}_{-0.04}$	$1.09^{+0.04}_{-0.04}$	$0.0049^{+0.0014}_{-0.0017}$	$28.4^{+0.4}_{-0.4}$	8.4	$0.86^{+0.07}_{-0.06}$	20.9	25.6
n	QUMOND local	$1.12^{+0.05}_{-0.05}$	$1.23^{+0.04}_{-0.04}$	$\mathit{0.0051}^{+0.0032}_{-0.0018}$	$29.1^{+0.4}_{-0.5}$	8.6	$0.82^{+0.07}_{-0.06}$	1080	1470

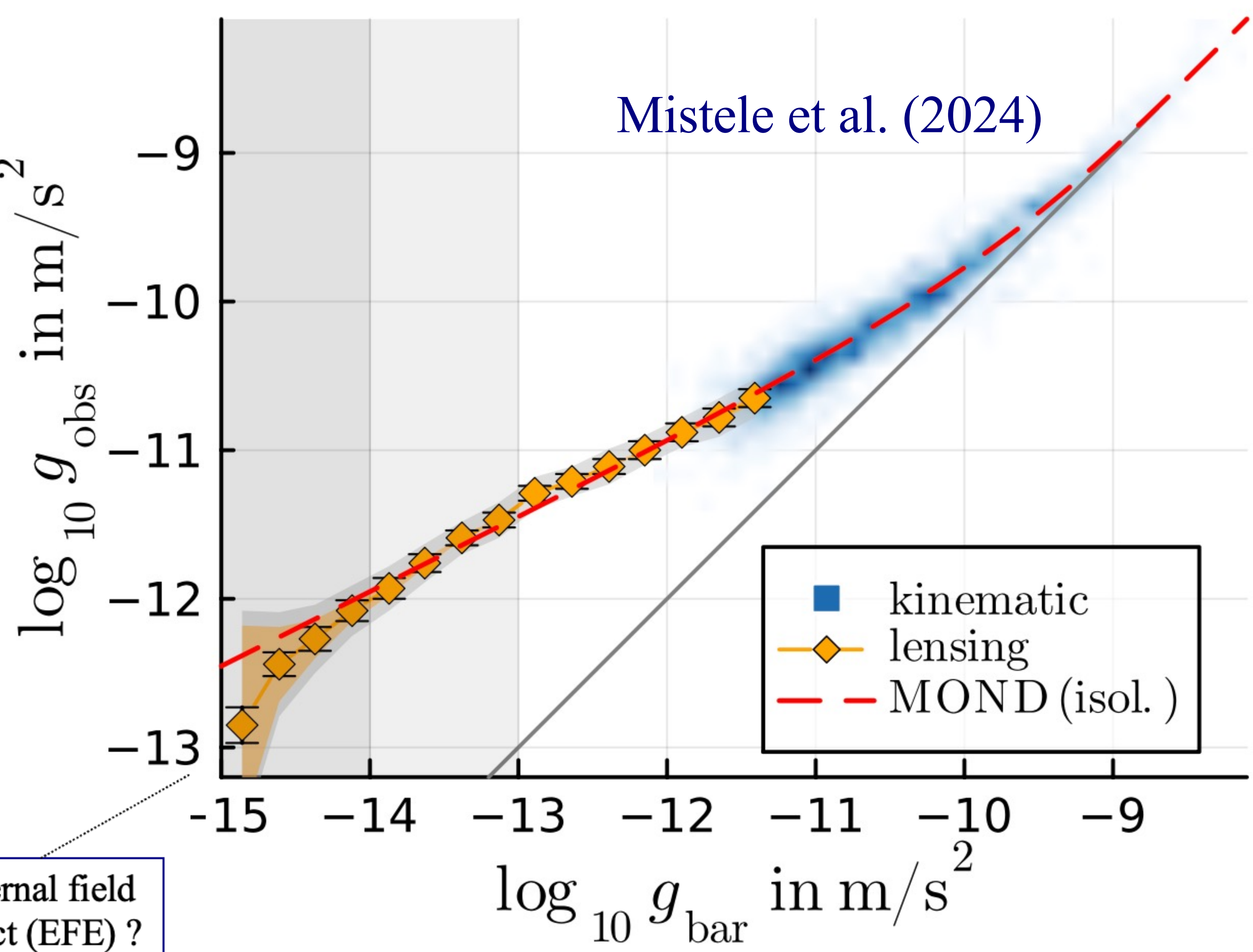
Individual galaxy rotation curves



No problem of diversity or BTFR tightness

$$\delta = \gamma = 1$$

Weak lensing

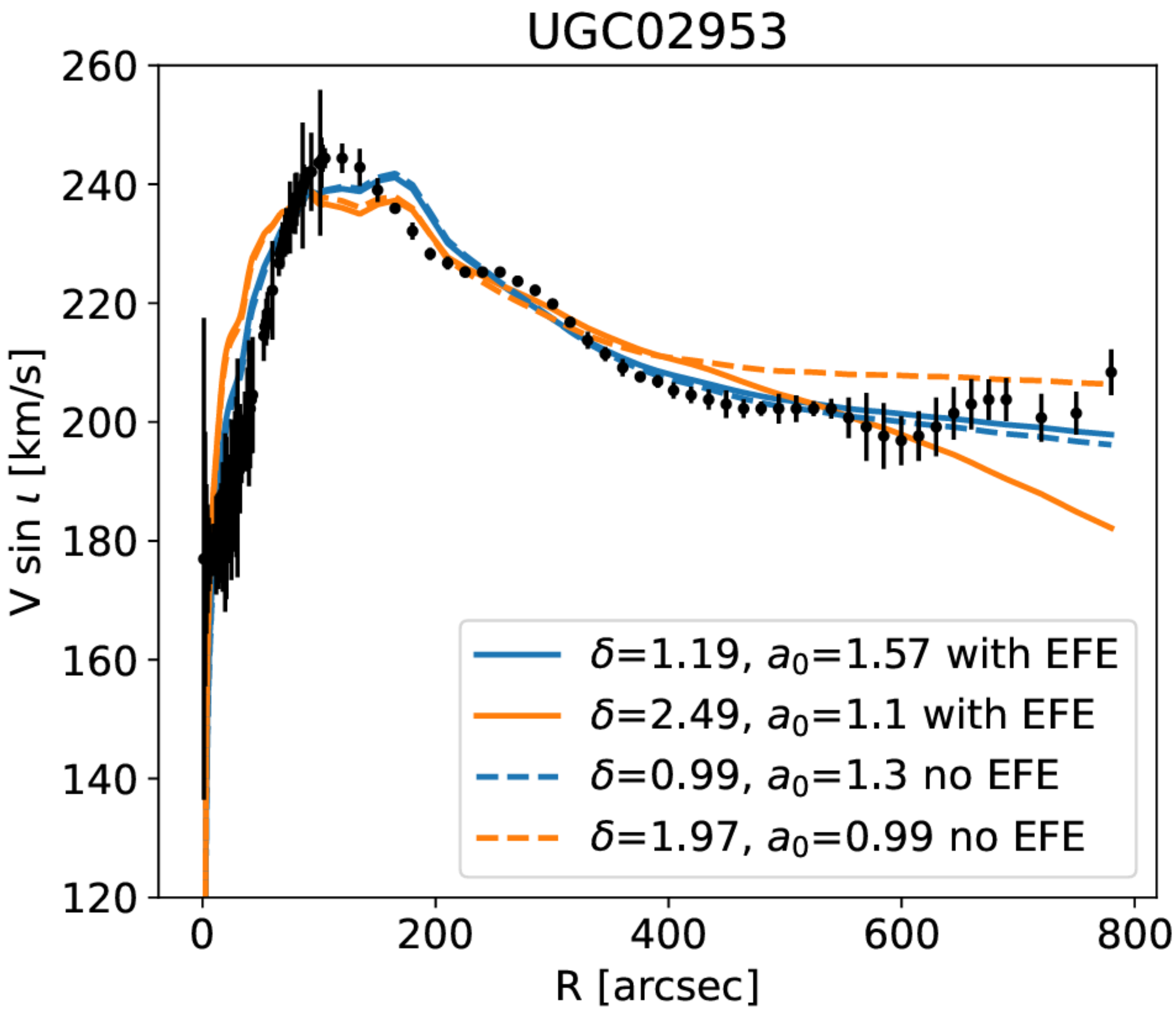


External field effect (EFE) ?

Hyperpriors on stellar M/L

M/L model	EFE model	shape $\boldsymbol{\delta}$	a_0	$\sigma_{\rm int}$	$\mu_{\rm d}$	$\mu_{\rm b}$	Q_2	σ_{Q2}	$\alpha_{\rm grav}$
Free hyper	No EFE	$1.28^{+0.06}_{-0.06}$	$1.04^{+0.03}_{-0.03}$	0.034	$0.71^{+0.03}_{-0.03}$	$0.62^{+0.04}_{-0.03}$	$25.5^{+0.8}_{-0.9}$	7.2	$0.40^{+0.05}_{-0.05}$
Free hyper	AQUAL local	$1.50^{+0.08}_{-0.07}$	$1.21^{+0.04}_{-0.04}$	0.032	$0.72^{+0.03}_{-0.03}$	$0.62^{+0.04}_{-0.03}$	$25.6^{+0.9}_{-0.9}$	7.2	$0.34^{+0.05}_{-0.05}$
Const hyper	No EFE	$1.24^{+0.06}_{-0.06}$	$1.05^{+0.03}_{-0.03}$	0.034	$0.68^{+0.02}_{-0.02}$	$0.69^{+0.02}_{-0.02}$	$26.3^{+0.7}_{-0.8}$	7.5	$0.44^{+0.05}_{-0.05}$
Const hyper	AQUAL local	$1.45^{+0.07}_{-0.07}$	$1.23^{+0.04}_{-0.04}$	0.032	$0.69^{+0.02}_{-0.02}$	$0.70^{+0.02}_{-0.02}$	$26.6^{+0.8}_{-0.9}$	7.6	$0.39^{+0.05}_{-0.05}$
Free unif	No EFE	$1.28^{+0.06}_{-0.06}$	$1.10^{+0.03}_{-0.03}$	0.031	$\mathit{0.73^{+0.72}_{-0.40}}$	$\mathit{0.68^{+0.41}_{-0.27}}$	$26.5^{+0.7}_{-0.8}$	7.6	$0.44^{+0.05}_{-0.05}$
Free unif	AQUAL local	$1.52^{+0.08}_{-0.07}$	$1.24^{+0.04}_{-0.04}$	0.029	$\mathit{0.74^{+0.70}_{-0.38}}$	$\mathit{0.60^{+0.53}_{-0.19}}$	$25.9^{+0.9}_{-0.9}$	7.3	$0.34^{+0.05}_{-0.05}$
Const unif	No EFE	$1.06^{+0.04}_{-0.04}$	$1.18^{+0.04}_{-0.04}$	0.032	$\mathit{0.58^{+0.73}_{-0.33}}$	$\mathit{0.73^{+0.58}_{-0.27}}$	$30.1^{+0.4}_{-0.5}$	8.9	$0.75^{+0.06}_{-0.06}$
Const unif	AQUAL local	$1.24^{+0.06}_{-0.05}$	$1.39^{+0.05}_{-0.05}$	0.031	$\mathit{0.57^{+0.72}_{-0.29}}$	$\mathit{0.74^{+0.63}_{-0.28}}$	$30.8^{+0.5}_{-0.6}$	9.1	$0.68^{+0.06}_{-0.06}$
No bulge	No EFE	$1.97^{+0.22}_{-0.18}$	$0.99^{+0.03}_{-0.03}$	0.041	$0.81^{+0.03}_{-0.03}$	—	$14.3^{+3.0}_{-2.9}$	2.7	$0.05^{+0.04}_{-0.04}$
No bulge	AQUAL local	$2.49^{+0.33}_{-0.27}$	$1.10^{+0.04}_{-0.04}$	0.039	$0.81^{+0.03}_{-0.03}$	—	$10.9^{+3.2}_{-2.9}$	1.9	$0.00^{+0.03}_{-0.02}$
Only bulge	No EFE	$0.99^{+0.06}_{-0.06}$	$1.30^{+0.08}_{-0.07}$	0.026	$0.59^{+0.05}_{-0.05}$	$0.68^{+0.04}_{-0.04}$	$31.7^{+0.5}_{-0.6}$	9.4	$0.99^{+0.13}_{-0.12}$
Only bulge	AQUAL local	$1.19^{+0.08}_{-0.07}$	$1.57^{+0.10}_{-0.09}$	0.024	$0.62^{+0.05}_{-0.04}$	$0.69^{+0.04}_{-0.04}$	$32.7^{+0.7}_{-0.8}$	9.6	$0.87^{+0.11}_{-0.10}$

Hyperpriors on stellar M/L



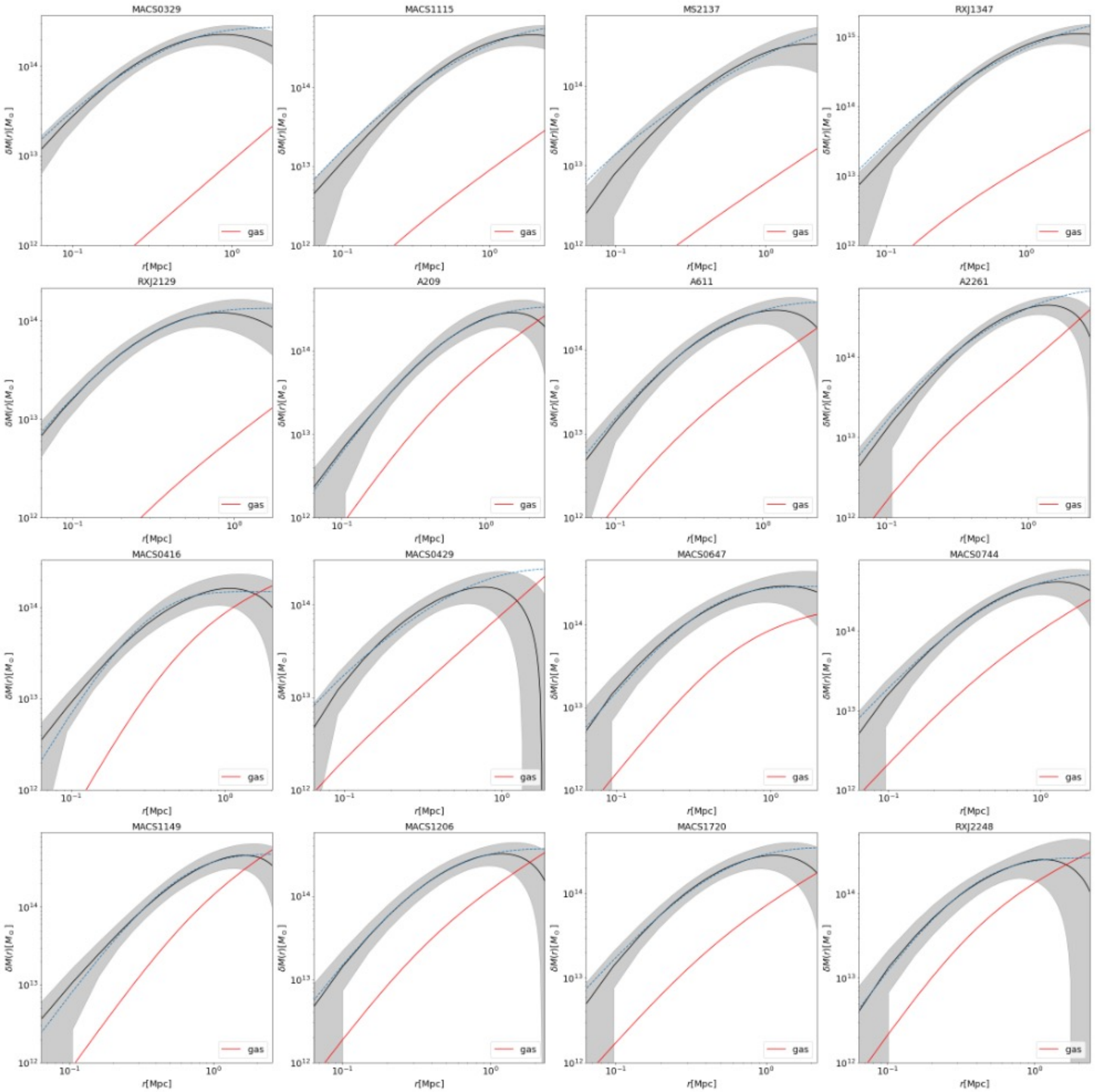
Modified gravity vs algebraic

$$a = \nu(a_N^+/a_0)\,a_{\rm N} \qquad \text{with} \qquad a_N^+ \equiv \left(a_N^2 + (2\pi G\Sigma)^2\right)^{1/2}$$

M/L model	shape (n)	a_0	$\sigma_{\rm int}$	$\mu_{\rm d}$	$\mu_{\rm b}$	Q_2	σ_{Q2}	$\alpha_{\rm grav}$	ΔBIC
Fiducial	$0.78^{+0.03}_{-0.03}$	$0.96^{+0.05}_{-0.05}$	0.046	0.5	0.7	$29.2^{+0.3}_{-0.2}$	8.7	$1.34^{+0.09}_{-0.08}$	2050
Free hyper	$1.57^{+0.09}_{-0.09}$	$1.12^{+0.03}_{-0.03}$	0.042	$0.97^{+0.03}_{-0.03}$	$0.64^{+0.04}_{-0.04}$	$22.8^{+1.0}_{-1.0}$	6.3	$0.33^{+0.05}_{-0.05}$	2290
Free unif	$1.34^{+0.08}_{-0.07}$	$1.10^{+0.04}_{-0.03}$	0.041	$\mathit{0.88}^{+0.76}_{-0.40}$	$\mathit{0.64}^{+0.43}_{-0.25}$	$25.2^{+0.8}_{-0.9}$	7.1	$0.49^{+0.06}_{-0.06}$	2120
No bulge	$1.93^{+0.19}_{-0.17}$	$1.01^{+0.03}_{-0.03}$	0.049	$1.08^{+0.04}_{-0.04}$	—	$17.5^{+2.4}_{-2.4}$	3.8	$0.15^{+0.05}_{-0.04}$	3680

Slight preference for algebraic relation
over modified gravity approximated correction

Galaxy clusters



$$\rho_{\rm res}(r) = \eta \rho_{\rm gas}(r) \exp(-r/r_{\rm cut})$$

in prep.

$\eta \simeq 10$ and $r_{\rm cut} \simeq 450$ kpc

Small scales

Object	Distance (AU)		a_p $(10^{-8}$ cm/s$^2)$
Mercury	0.39		0.04
Icarus	1.08		6.3
Mars	1.52	$\longrightarrow$ Maximal anomalous acceleration allowed	0.1
Jupiter	5.2		0.12
Uranus	19.2		0.08*
Neptune	30.1		0.13*

Sanders (2006)

$$\boxed{\mu\left(\frac{g}{a_0}\right)\times g = g_N \qquad \text{or} \qquad \nu\left(\frac{g_N}{a_0}\right)\times g_N = g}$$

$$\mu(x) = \frac{x}{1+x}$$

Ruled out

$$\nu(x) = \frac{1}{1-\exp(-\sqrt{x})}$$

Allowed

Small scales

Gravitational field of the Milky Way acting on Solar neighbourhood ~ **1.8 x a_0**

For local Wide Binaries with mutual attraction below a_0
***n*=1 =>** $(\langle g_r \rangle / g_N)^{1/2} \sim 1.2$
hence a **20% enhancement** of relative velocities

In the Solar System, AQUAL/QUMOND also creates a quadrupole field (***e*** pointing towards the Galactic center) :

$$\Phi = -\frac{GM}{r} - \frac{Q_2}{2} x^i x^j \left(e_i e_j - \frac{1}{3}\delta_{ij} \right)$$

Q2 (in s^{-2}) can be constrained at Saturn (Cassini data)

Wide Binaries

Gaia DR3 provides parallaxes, prop. motions and v_{los} for ~3.3 x 10^7 stars

Isolate ~10^4 wide binaries (within 300 pc from the Sun, mass ranging from 0.2 to 1.3 M_{sun}) with separations up to ~0.15 pc

Model their relative velocities with a 6 parameters model :

$$P(a) \propto \begin{cases} a^{-1}, & \text{if } a < a_{\rm break} \\ a^{\beta}, & \text{if } a > a_{\rm break} \end{cases} \qquad P(e) = (\gamma + 1)\, e^{\gamma}$$

+ fraction of close binary contamination (triples) + the CBs maximum semi-major axis + line-of-sight Galactic contamination

=> Likelihood ratio favours Newton over ***n=1*** MOND **interpolating function** by orders of magnitude (Banik et al. 2024) but high fraction of triples (close to 70%) … **very hard to draw definitive conclusions !**

Solar System quadrupole

Cassini (Hees et al. 2014) :

$$Q_2 = (3 \pm 3) \times 10^{-27}\ \mathrm{s}^{-2}$$

$$\alpha_{\rm grav} \equiv \frac{\sqrt{\eta}-1}{0.193},$$

where $\eta \equiv \langle g_r \rangle / g_{\rm N}$

Wide Binaries

(Banik et al. 2024)

M/L model	EFE model	shape $\boldsymbol{\delta}$	a_0	$\sigma_{\rm int}$	$\mu_{\rm d}$	$\mu_{\rm b}$	Q_2	σ_{Q2}	$\alpha_{\rm grav}$
Free hyper	No EFE	$1.28^{+0.06}_{-0.06}$	$1.04^{+0.03}_{-0.03}$	0.034	$0.71^{+0.03}_{-0.03}$	$0.62^{+0.04}_{-0.03}$	$25.5^{+0.8}_{-0.9}$	7.2	$0.40^{+0.05}_{-0.05}$
Free hyper	AQUAL local	$1.50^{+0.08}_{-0.07}$	$1.21^{+0.04}_{-0.04}$	0.032	$0.72^{+0.03}_{-0.03}$	$0.62^{+0.04}_{-0.03}$	$25.6^{+0.9}_{-0.9}$	7.2	$0.34^{+0.05}_{-0.05}$
Const hyper	No EFE	$1.24^{+0.06}_{-0.06}$	$1.05^{+0.03}_{-0.03}$	0.034	$0.68^{+0.02}_{-0.02}$	$0.69^{+0.02}_{-0.02}$	$26.3^{+0.7}_{-0.8}$	7.5	$0.44^{+0.05}_{-0.05}$
Const hyper	AQUAL local	$1.45^{+0.07}_{-0.07}$	$1.23^{+0.04}_{-0.04}$	0.032	$0.69^{+0.02}_{-0.02}$	$0.70^{+0.02}_{-0.02}$	$26.6^{+0.8}_{-0.9}$	7.6	$0.39^{+0.05}_{-0.05}$
Free unif	No EFE	$1.28^{+0.06}_{-0.06}$	$1.10^{+0.03}_{-0.03}$	0.031	$\mathit{0.73}^{+0.72}_{-0.40}$	$\mathit{0.68}^{+0.41}_{-0.27}$	$26.5^{+0.7}_{-0.8}$	7.6	$0.44^{+0.05}_{-0.05}$
Free unif	AQUAL local	$1.52^{+0.08}_{-0.07}$	$1.24^{+0.04}_{-0.04}$	0.029	$\mathit{0.74}^{+0.70}_{-0.38}$	$\mathit{0.60}^{+0.53}_{-0.19}$	$25.9^{+0.9}_{-0.9}$	7.3	$0.34^{+0.05}_{-0.05}$
Const unif	No EFE	$1.06^{+0.04}_{-0.04}$	$1.18^{+0.04}_{-0.04}$	0.032	$\mathit{0.58}^{+0.73}_{-0.33}$	$\mathit{0.73}^{+0.58}_{-0.27}$	$30.1^{+0.4}_{-0.5}$	8.9	$0.75^{+0.06}_{-0.06}$
Const unif	AQUAL local	$1.24^{+0.06}_{-0.05}$	$1.39^{+0.05}_{-0.05}$	0.031	$\mathit{0.57}^{+0.72}_{-0.29}$	$\mathit{0.74}^{+0.63}_{-0.28}$	$30.8^{+0.5}_{-0.6}$	9.1	$0.68^{+0.06}_{-0.06}$
No bulge	No EFE	$1.97^{+0.22}_{-0.18}$	$0.99^{+0.03}_{-0.03}$	0.041	$0.81^{+0.03}_{-0.03}$	—	$14.3^{+3.0}_{-2.9}$	2.7	$0.05^{+0.04}_{-0.04}$
No bulge	AQUAL local	$2.49^{+0.33}_{-0.27}$	$1.10^{+0.04}_{-0.04}$	0.039	$0.81^{+0.03}_{-0.03}$	—	$10.9^{+3.2}_{-2.9}$	1.9	$0.00^{+0.03}_{-0.02}$
Only bulge	No EFE	$0.99^{+0.06}_{-0.06}$	$1.30^{+0.08}_{-0.07}$	0.026	$0.59^{+0.05}_{-0.05}$	$0.68^{+0.04}_{-0.04}$	$31.7^{+0.5}_{-0.6}$	9.4	$0.99^{+0.13}_{-0.12}$
Only bulge	AQUAL local	$1.19^{+0.08}_{-0.07}$	$1.57^{+0.10}_{-0.09}$	0.024	$0.62^{+0.05}_{-0.04}$	$0.69^{+0.04}_{-0.04}$	$32.7^{+0.7}_{-0.8}$	9.6	$0.87^{+0.11}_{-0.10}$

Desmond, Hees & Famaey (2024)

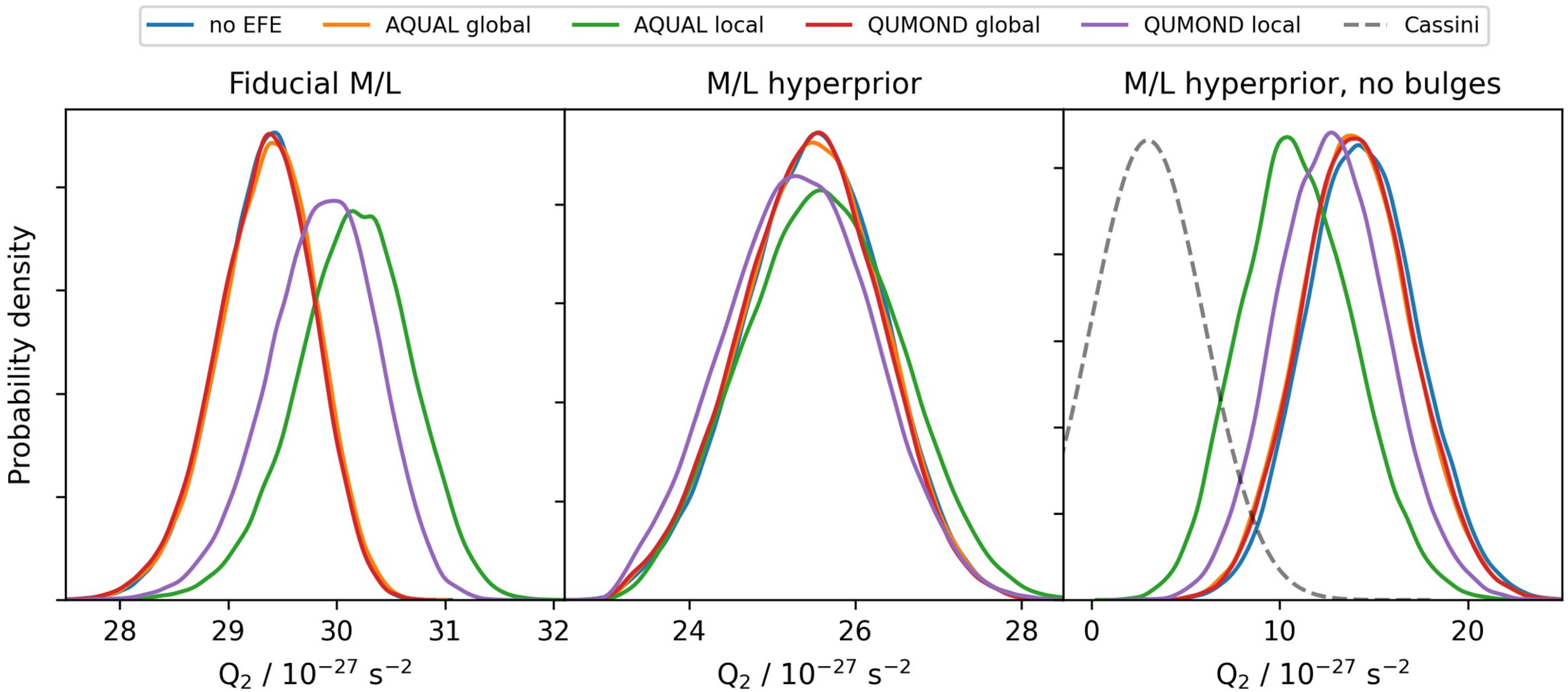
Solar System quadrupole

Cassini (Hees et al. 2014) :

$$Q_2 = (3 \pm 3) \times 10^{-27}\ \mathrm{s}^{-2}$$

$$\alpha_{\mathrm{grav}} \equiv \frac{\sqrt{\eta} - 1}{0.193},$$

where $\eta \equiv \langle g_r \rangle / g_{\mathrm{N}}$



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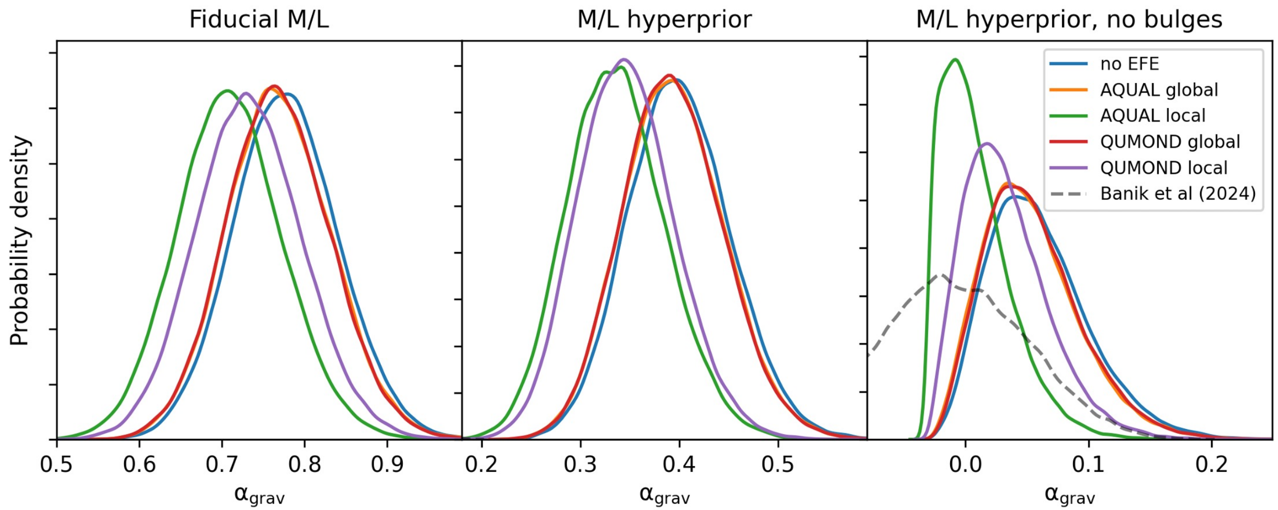

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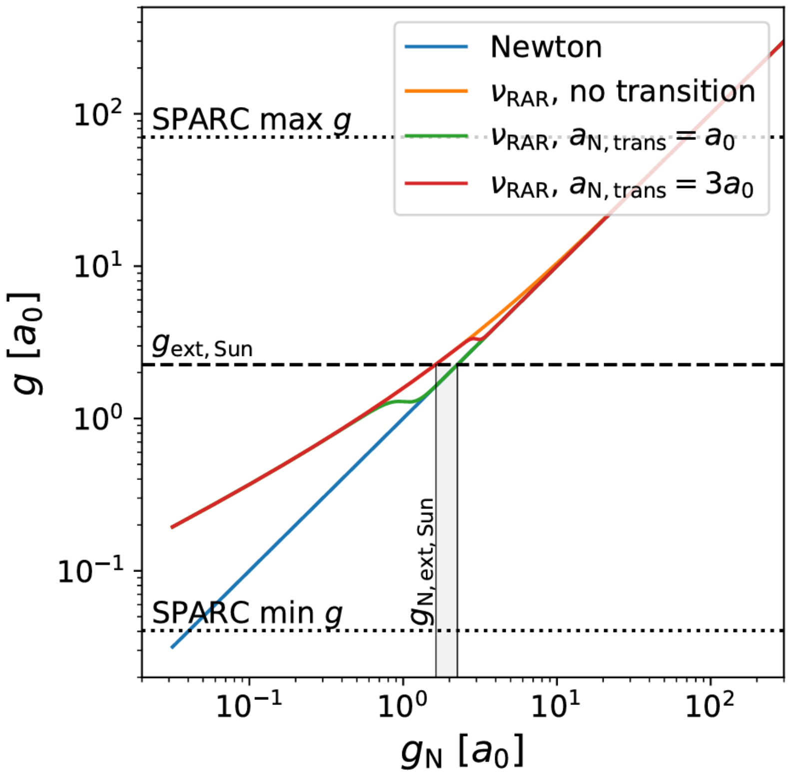
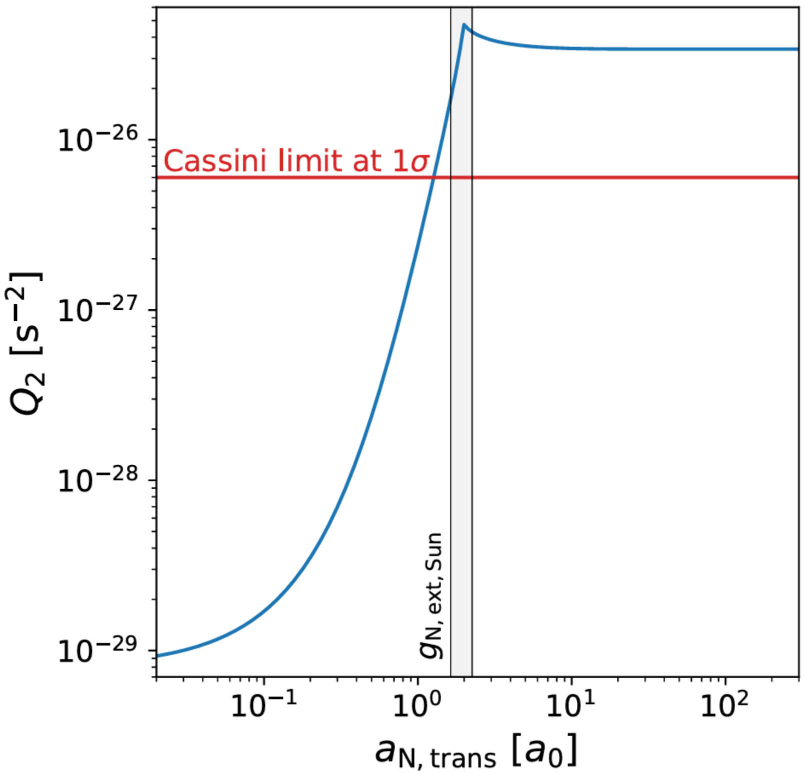




Desmond, Hees & Famaey (2024)

Cosmology ?

Classical action: $S_{\text{grav BM}} \equiv -\int \frac{a_0^2 F(|\nabla\Phi|^2/a_0^2)}{8\pi G} d^3x\, dt$

⇒ Add some k-essence like scalar + a vector field for lensing

⇒ Latest version by Skordis & Zlosnik (2021), « AEST »:

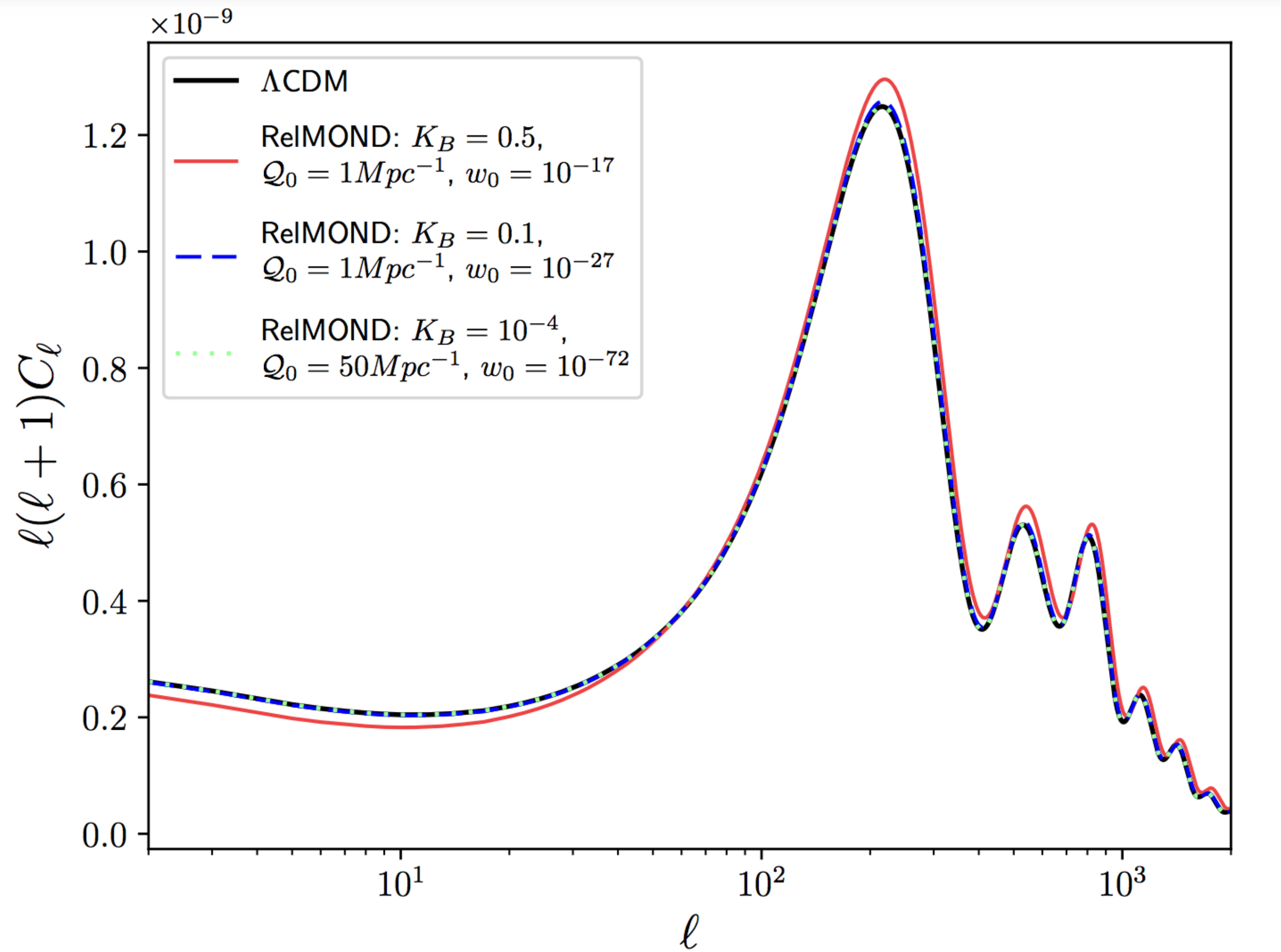
$$\mathcal{Q} = A^\mu \nabla_\mu \phi \longrightarrow \dot{\phi}$$

$$\mathcal{Y} = \mathcal{Q}^2 + (\nabla\phi)^2 \longrightarrow |\vec{\nabla}\varphi|^2$$

$$\boxed{\mathcal{F} = \underbrace{-2\mathcal{K}_2\left(\mathcal{Q} - \mathcal{Q}_0\right)^2}_{\text{"dust" cosmology}} + \underbrace{(2 - K_\mathrm{B})\mathcal{Y}}_{\text{Mixing}} + \underbrace{\frac{2(2-K_\mathrm{B})}{3a_0}\mathcal{Y}^{3/2}}_{\text{MOND}} + \ldots}$$

Bonus: GW and light speeds are equal

Cosmology ?



Skordis & Zlosnik (2021)

Conclusion

Baryonic Tully-Fisher relation between baryonic mass of spiral galaxies and asymptotic velocity is captured by Milgrom's relation, while the high-end slope and the scatter remain challenging in ΛCDM

The diversity of RC shapes driven by the surface density of baryons is also captured by Milgrom's relation, and remains challenging for simulations that either produce too few or too many cores

MOND is successful at predicting the dynamics of **galaxies**, especially rotationally-supported ones: the question is **why** does it make successful predictions ?

- Emergence in ΛCDM?
- Fundamental nature of DM?
- Modified gravity?
- All of the above or something even more exotic?

The **interpolating function and or EFE** seem to need to be scale-dependent